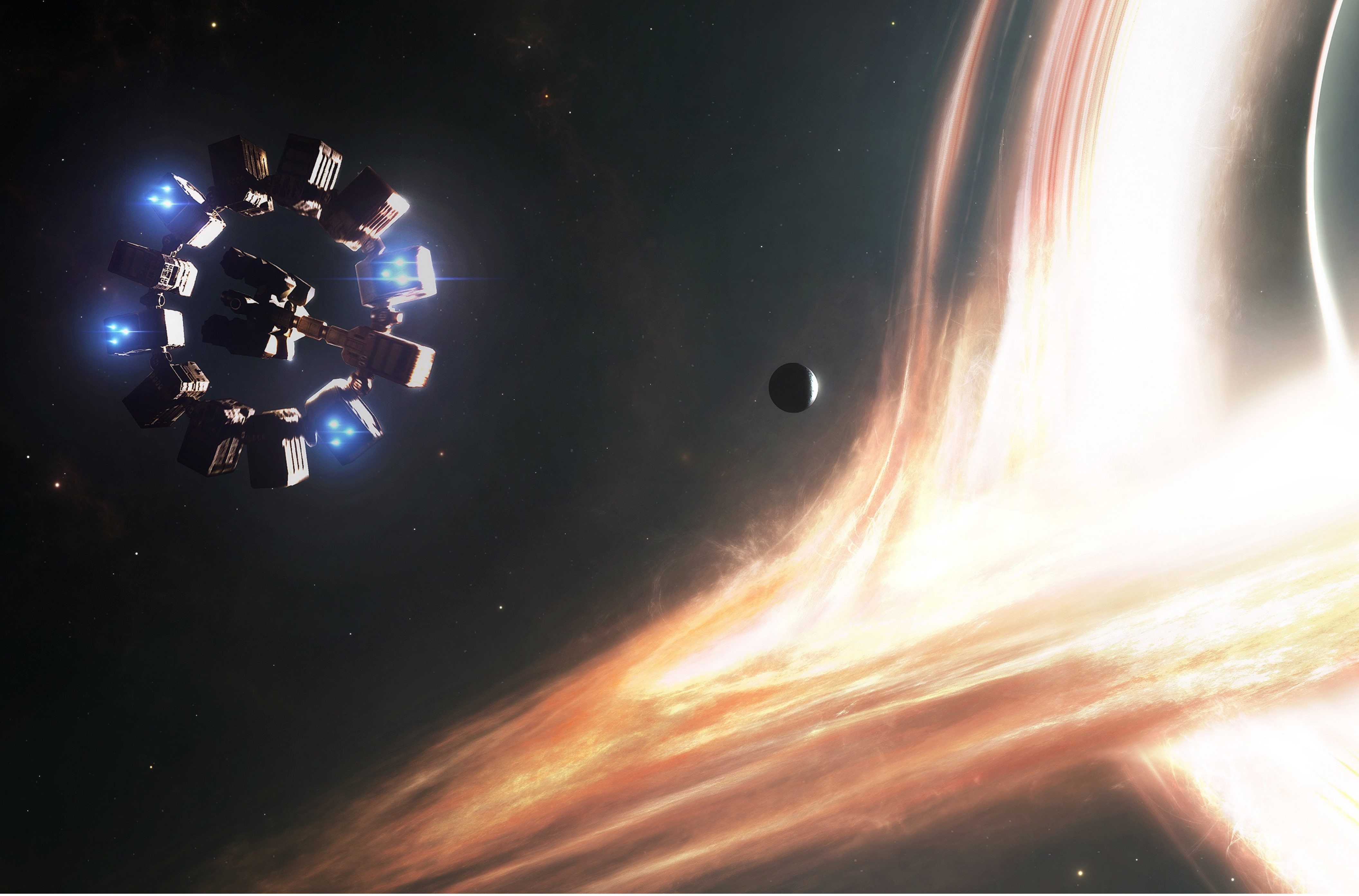
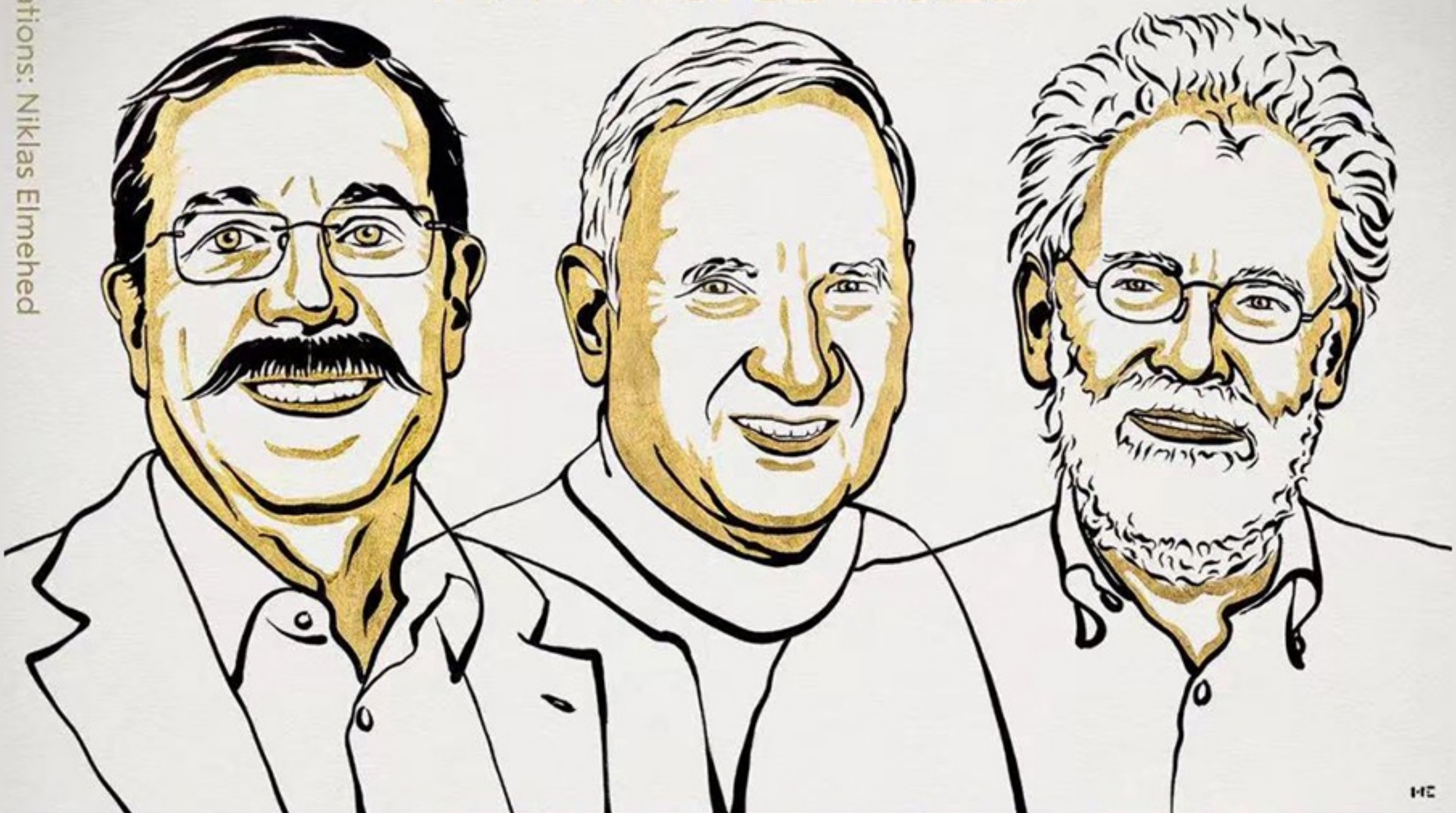


Quantum Algorithms II

THE NOBEL PRIZE IN PHYSICS 2022

Illustrations: Niklas Elmehed



Alain Aspect

John F. Clauser

Anton Zeilinger

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

THE ROYAL SWEDISH ACADEMY OF SCIENCES

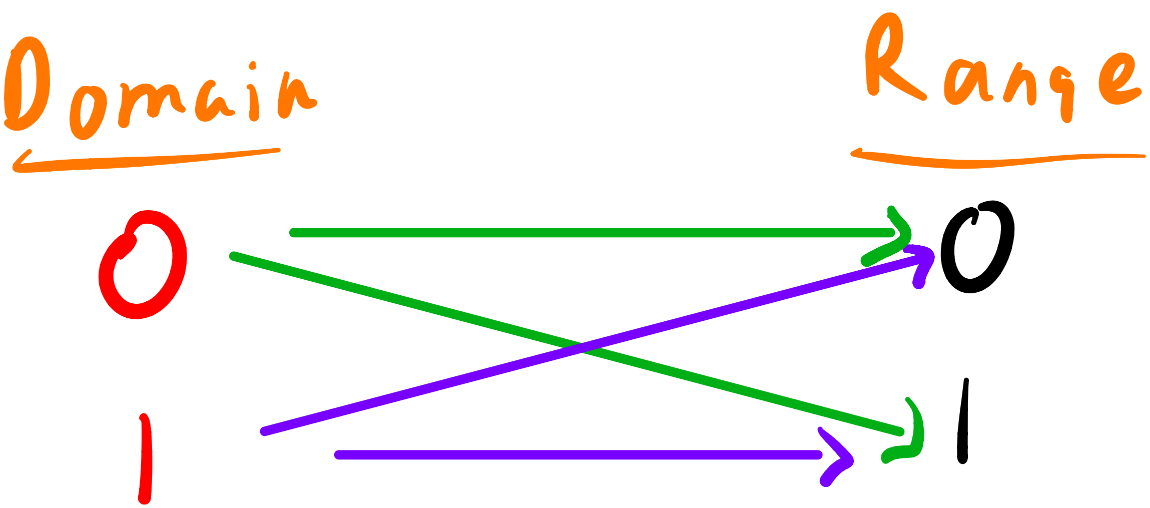
The Quantum Computing Stack

5	Applications
3	Algorithms
2	Circuits
4	Hardware
1	Mechanics

You are Here!

Contrived Problem

Difficult to compute $f(x)$



$f(0) = 0$ or 1

$f(1) = 0$ or 1

Want to know

Does $f(0) = f(1)$?

evaluate evaluate

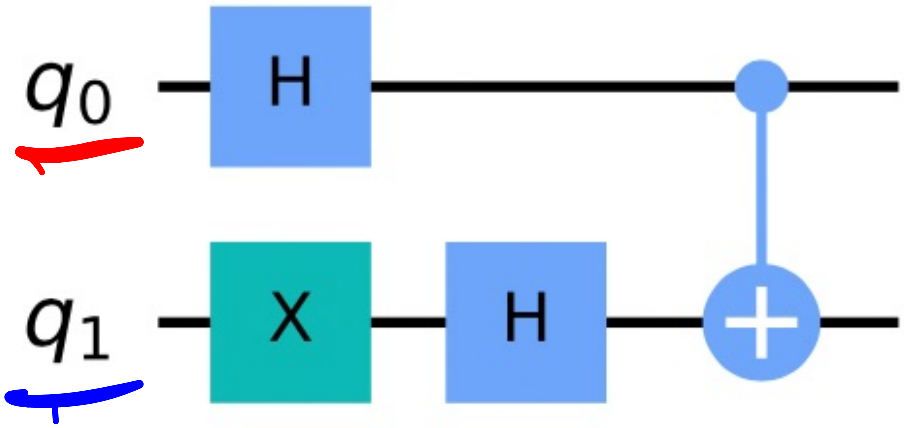
Classical Approach

2 evaluations

Deutch's Algorithm

1 evaluation

Phase Kickback



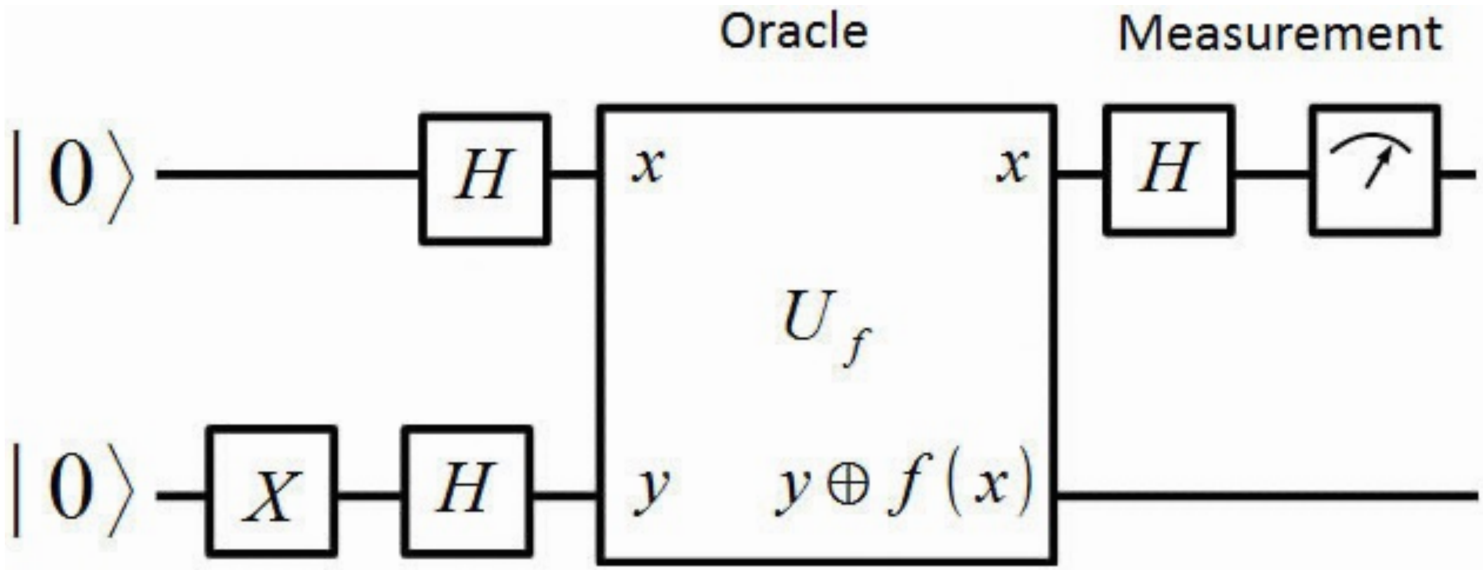
$$|q_1 q_0\rangle = |-+\rangle$$

$$\mathrm{CNOT}\,|-+\rangle = |--\rangle$$

Control Phase changes

Action state changes

Deutch's Algorithm



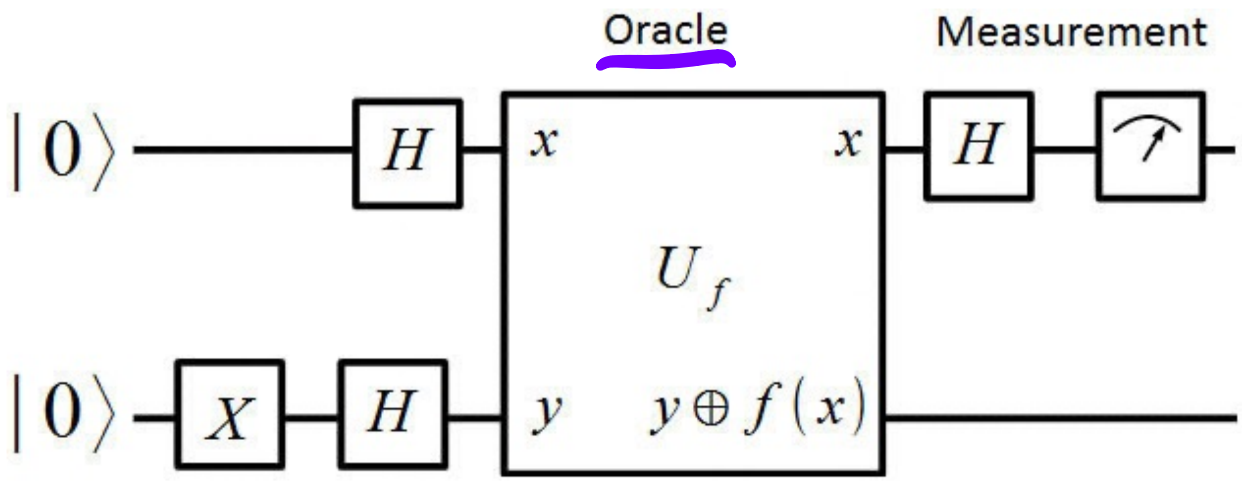
4. Measure

1. Super-position

2. Phase Kickback Result

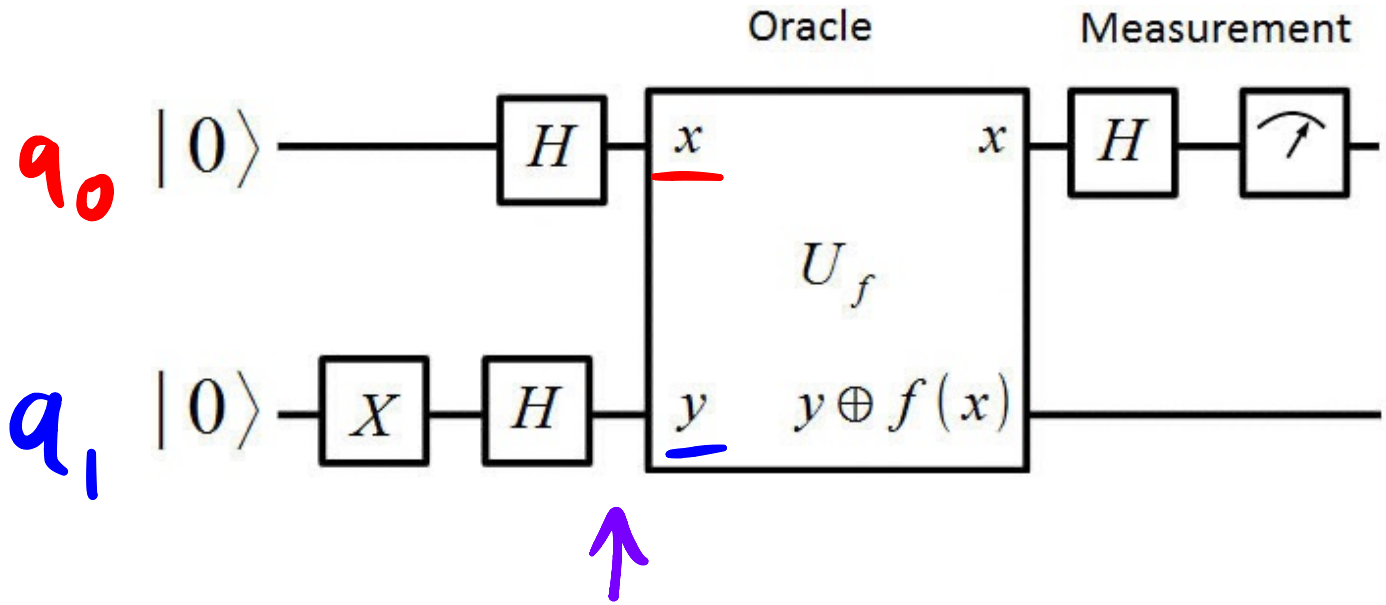
3. Phase -> State

Deutch's Algorithm



In just 1 call to f using our oracle, we find if

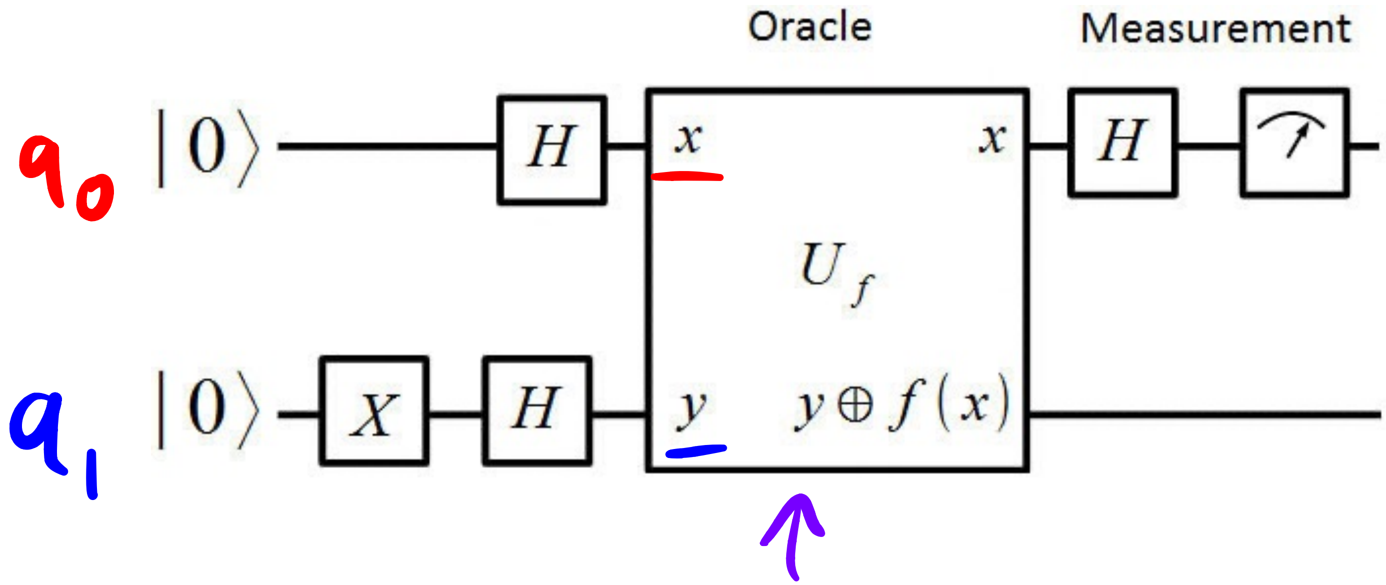
$f(0) = f(1)$ or $f(0) \neq f(1)$



$$|q_0\rangle = H|0\rangle = |+\rangle$$

$$|q_1\rangle = HX|0\rangle = H|1\rangle = |-\rangle$$

$$|q_1 q_0\rangle = |-+\rangle = \frac{|-\rangle|0\rangle + |-\rangle|1\rangle}{\sqrt{2}}$$



$$|q_1 q_0\rangle = |-+\rangle = \frac{|-\rangle|0\rangle + |-\rangle|1\rangle}{\sqrt{2}}$$

$$|q_1 q_0\rangle = U_f|-+\rangle \qquad U_f|-\rangle|0\rangle \qquad U_f|-\rangle|1\rangle$$

$$U_f|y\rangle|x\rangle = |y \oplus f(x)\rangle|x\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|\underline{- \oplus f(0)}\rangle|0\rangle + |\underline{- \oplus f(1)}\rangle|1\rangle\right)$$

$$|- \oplus f(0)\rangle = \frac{1}{\sqrt{2}}\left(|0 \oplus f(0)\rangle - |1 \oplus f(0)\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

if $f(0) = 0$

$$|- \oplus 0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$= +|-\rangle$$

if $f(0) = 1$

$$|- \oplus 1\rangle = \frac{1}{\sqrt{2}}\left(|0 \oplus 1\rangle - |1 \oplus 1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|1\rangle - |0\rangle\right)$$

$$= -|-\rangle$$

$|- \oplus f(0)\rangle$ $\quad$ $|- \oplus f(1)\rangle$

if $f(0) = 0$

$$|- \oplus 0\rangle = +|-\rangle$$

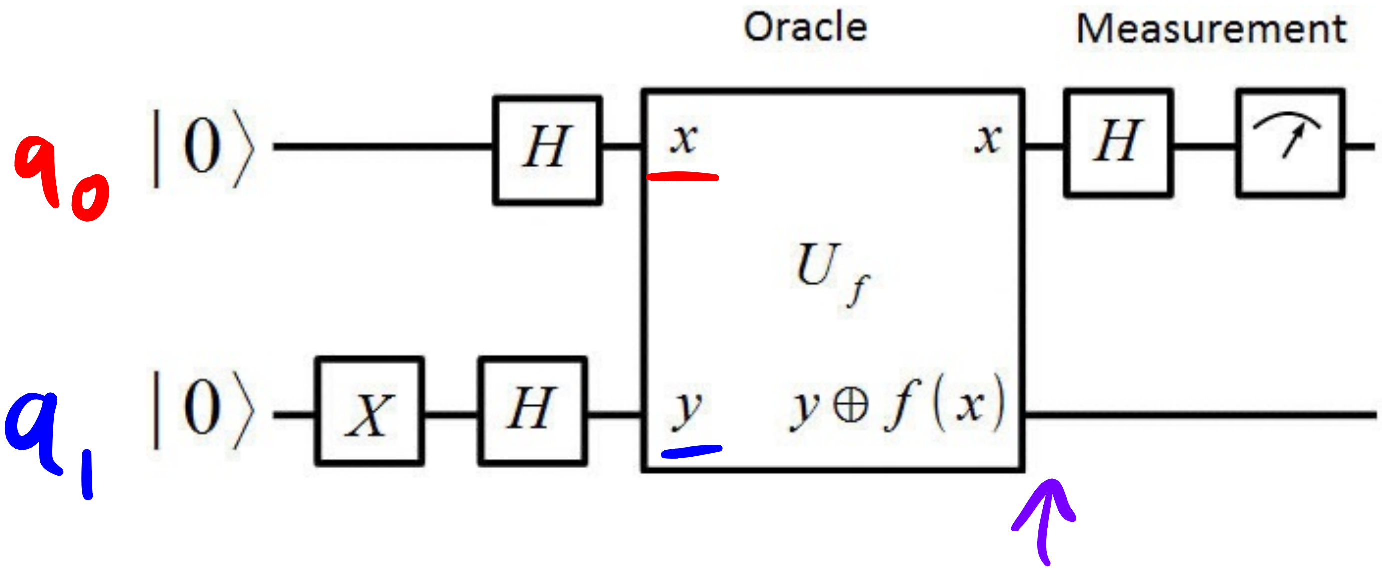
if $f(0) = 1$

$$|- \oplus 1\rangle = -|-\rangle$$

In general

$$|- \oplus f(0)\rangle = (-1)^{f(0)}|-\rangle$$

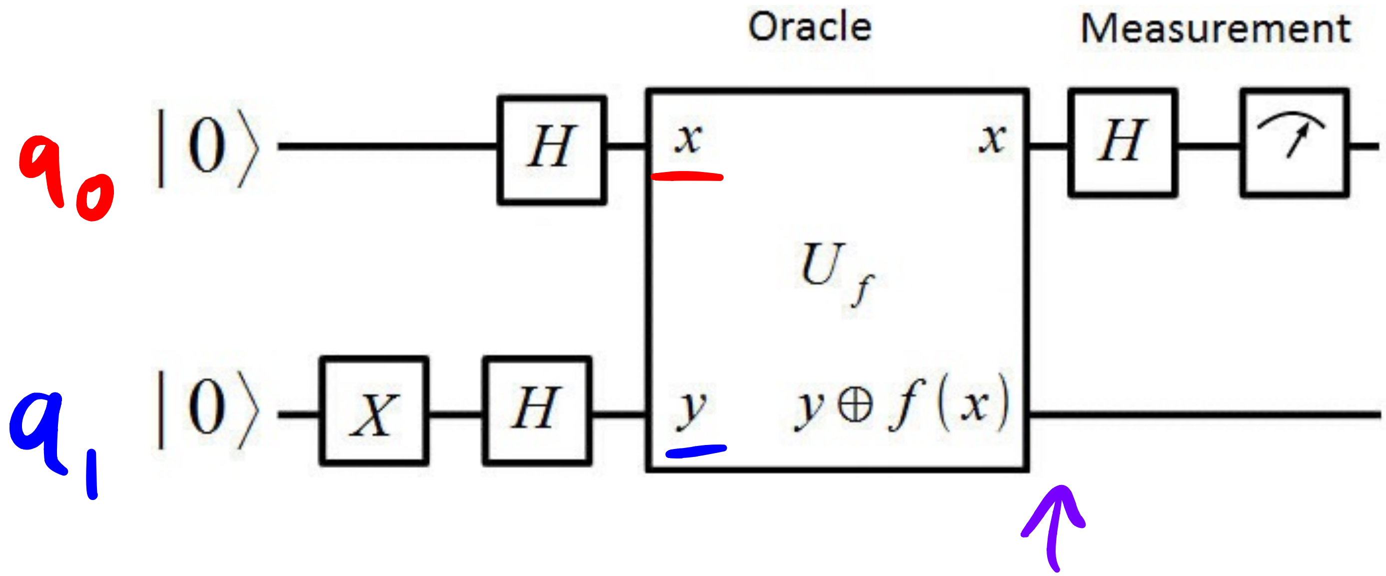
$$|- \oplus f(1)\rangle = (-1)^{f(1)}|-\rangle$$



$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}\left(\underbrace{|- \oplus f(0)\rangle}|0\rangle + \underbrace{|- \oplus f(1)\rangle}|1\rangle\right)$$

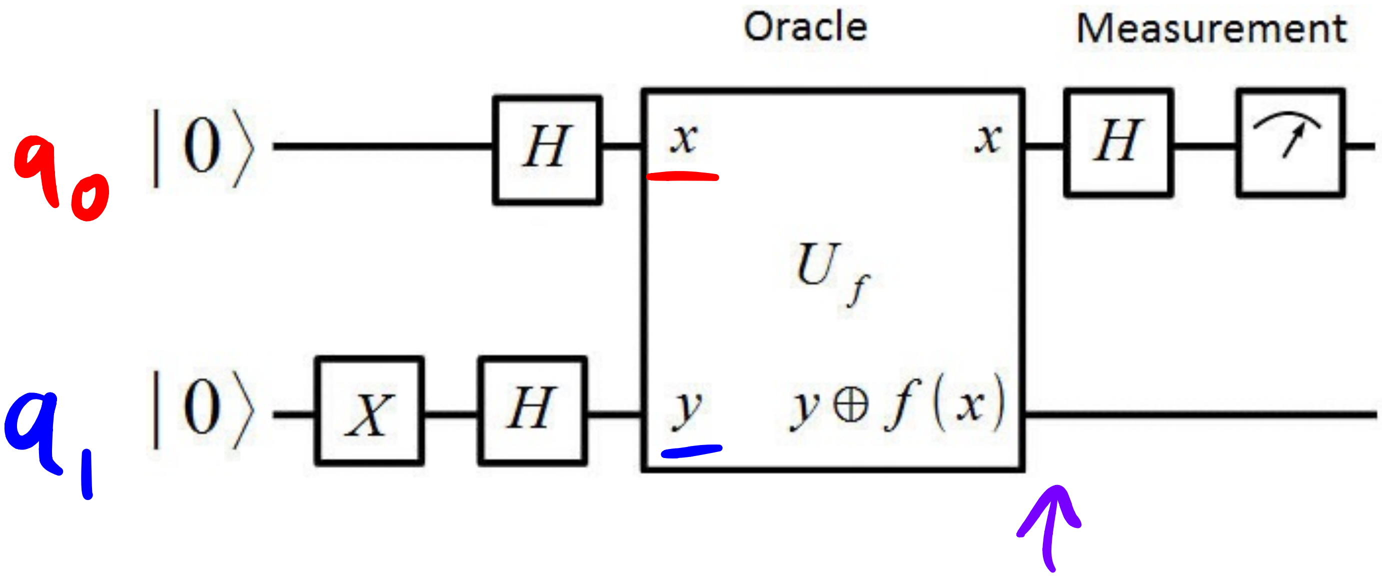
$$\boxed{\begin{aligned} |- \oplus f(0)\rangle &= (-1)^{f(0)}|-\rangle \\ |- \oplus f(1)\rangle &= (-1)^{f(1)}|-\rangle \end{aligned}}$$

$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}\left[\underbrace{(-1)^{f(0)}|-\rangle}|0\rangle + \underbrace{(-1)^{f(1)}|-\rangle}|1\rangle\right]$$



$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}\left[(-1)^{f(0)}|-\rangle|0\rangle + (-1)^{f(1)}|-\rangle|1\rangle\right]$$

$$= |-\rangle \otimes \frac{1}{\sqrt{2}}\left[(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right]$$



$$|q_1 q_0\rangle = |-\rangle \otimes \left[\frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}} \right]$$

$$|q_1\rangle = |-\rangle \qquad |q_0\rangle = \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}$$

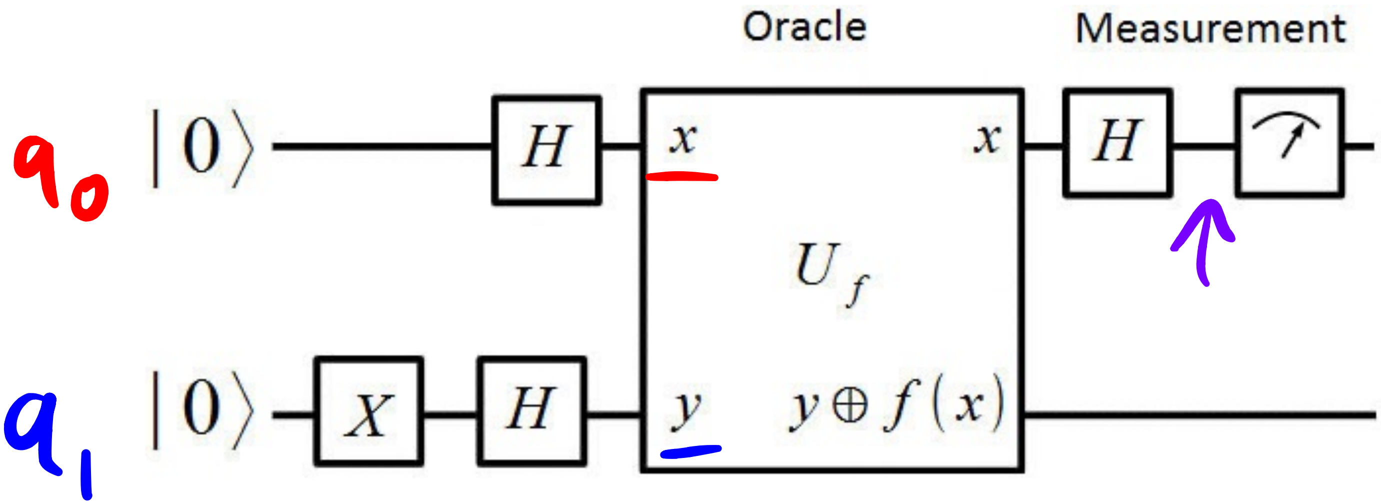
$$|q_0\rangle = \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}$$

$$f(0) = 0$$

$$f(1) = 1$$

$$\frac{(-1)^{0}|0\rangle + (-1)^{1}|1\rangle}{\sqrt{2}} = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$= |-\rangle$$



If $f(0) = f(1)$

Same

$$|q_0\rangle = \frac{\pm|0\rangle + \pm|1\rangle}{\sqrt{2}}$$

$$= \pm|+\rangle$$

$$H|q_0\rangle = \pm|0\rangle$$

If $f(0) \neq f(1)$

Different

$$|q_0\rangle = \frac{\pm|0\rangle + \mp|1\rangle}{\sqrt{2}}$$

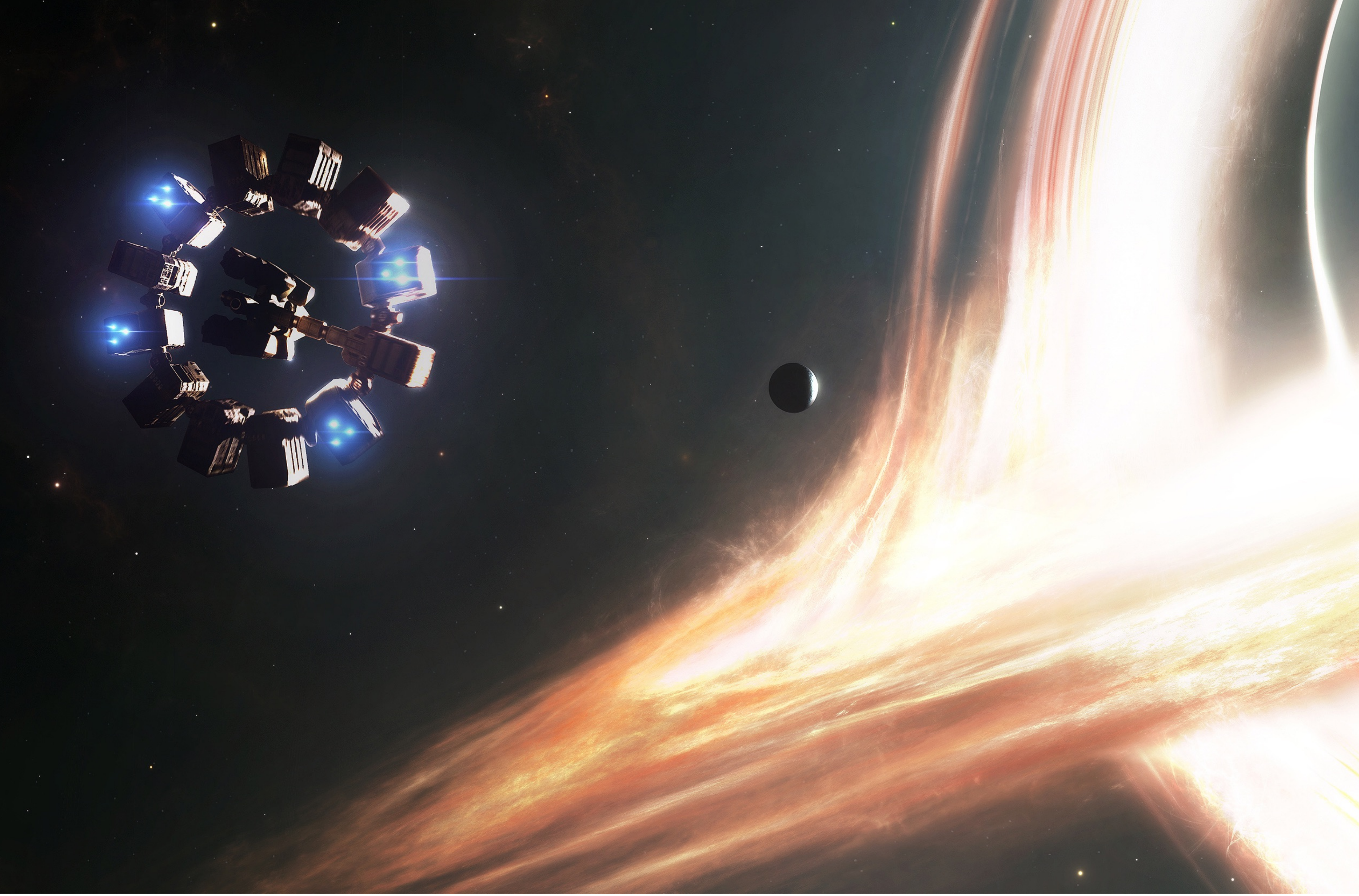
$$= \pm|-\rangle$$

$$H|q_0\rangle = \pm|1\rangle$$

$-|0\rangle$

$$\left(-\langle 0|\right)\left(-|0\rangle\right)$$

$$\begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = 1$$



Keyword Interstellar

Deutch-Josza

$$f \quad \underbrace{\{0,1\}}_{x}{}^{N} \to \{0,1\}$$

$|\cancel{x}| = 2^n$

ex. $X = 011\,01$ (101 01)

Balanced

for half of x

$f(x) = 0$

other half

$f(x) = 1$

Constant

all

$f(x) = 0$

or

$f(x) = 1$

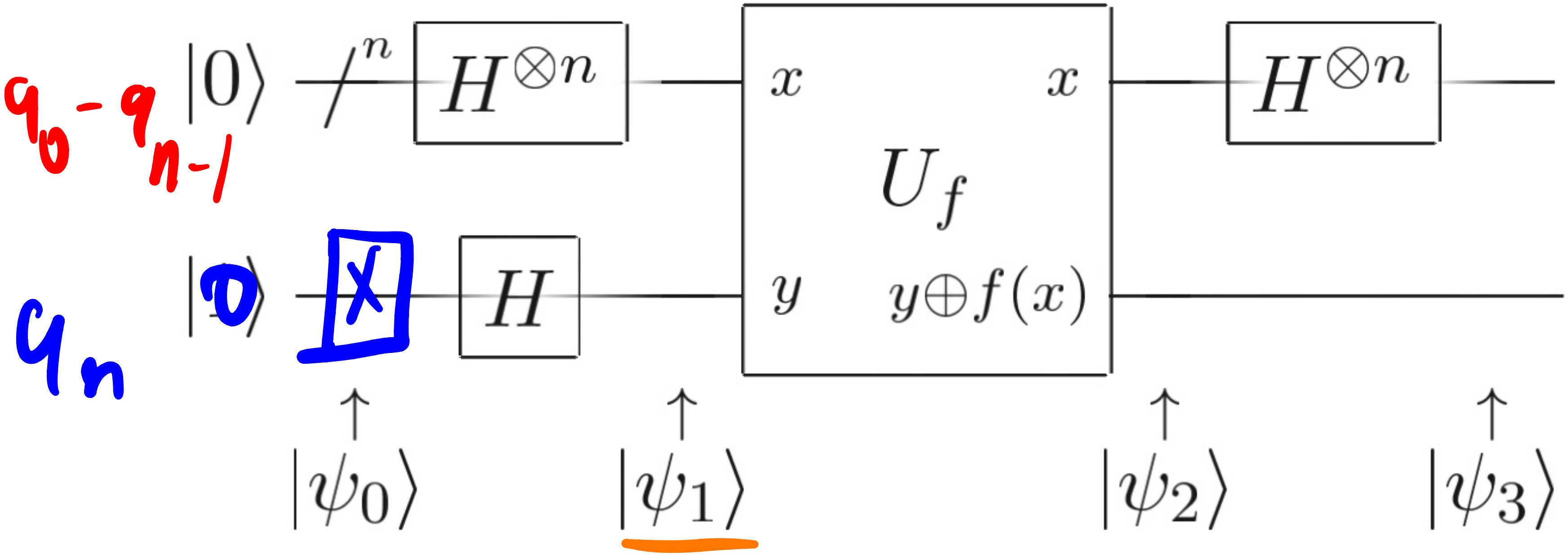
Deutch - Josza

Classical

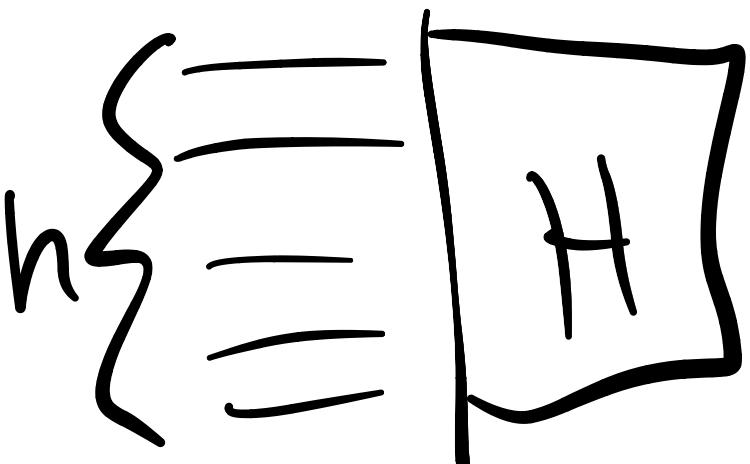
$O(2^N)$

Quantum

$O(1)$



$$|\psi_1\rangle = |-\rangle \otimes |+\rangle^{\otimes n}$$



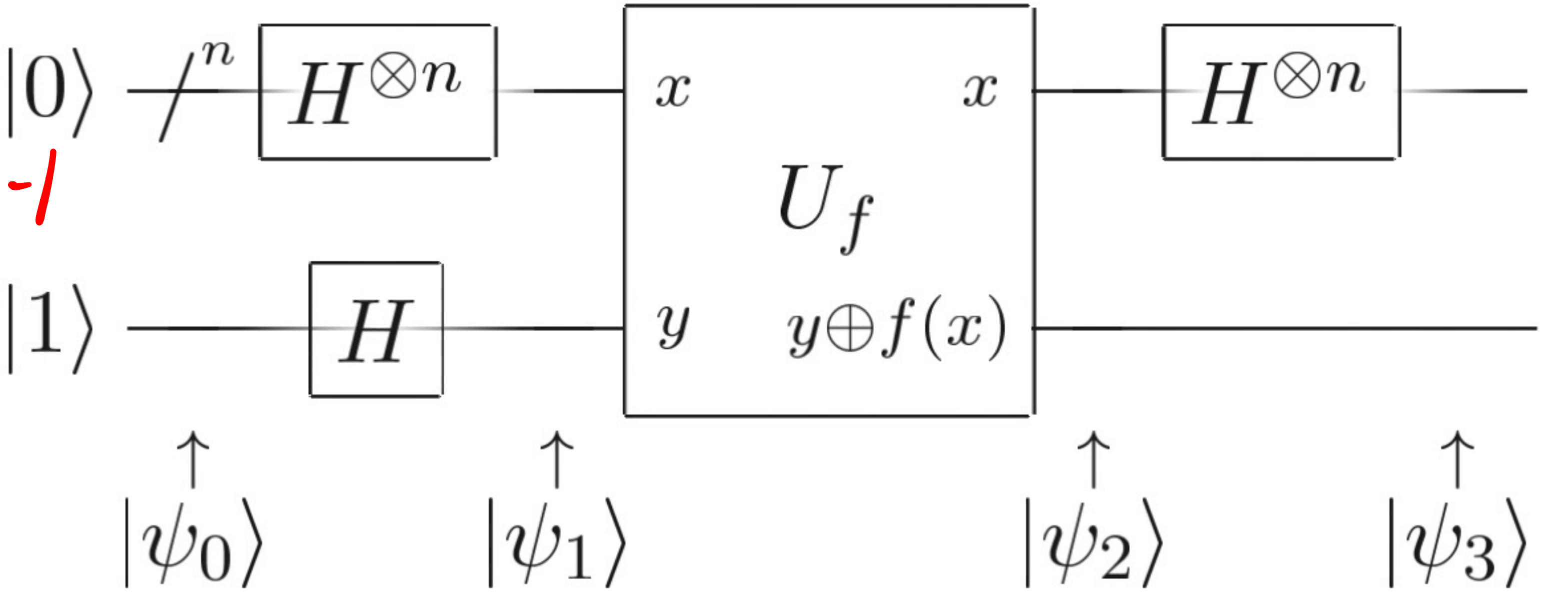
$$|+\rangle^{\otimes n} = \left.\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\right\} 2^n$$

n+1 qubit

0110 1010

$q_0 - q_{n-1}$

q_n



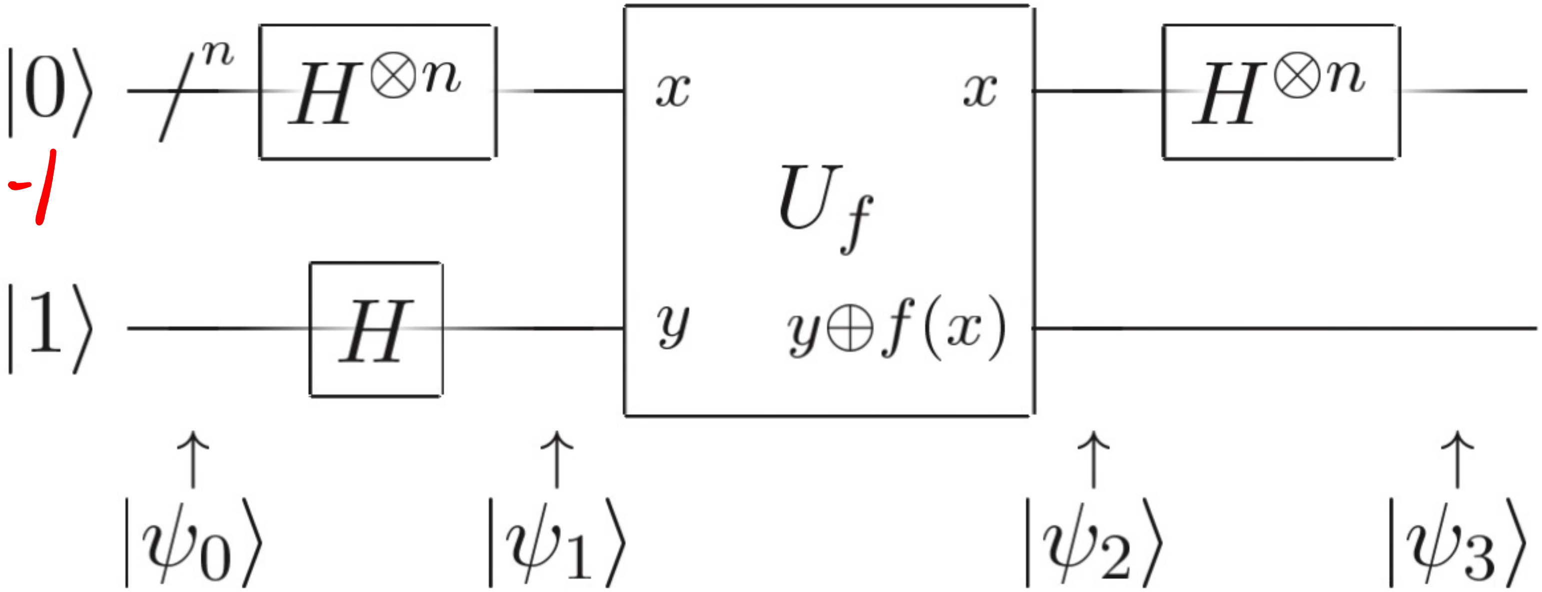
$$|\psi_2\rangle = |-\rangle \otimes \frac{1}{\sqrt{2^n}} \sum_x (-1)^{f(x)} |x\rangle$$

2-qubit

$$|-\rangle \otimes \left[\frac{(-1)^{f(0)} |0\rangle + (-1)^{f(1)} |1\rangle}{\sqrt{2}} \right]$$

$q_0 - q_{n-1}$

q_n

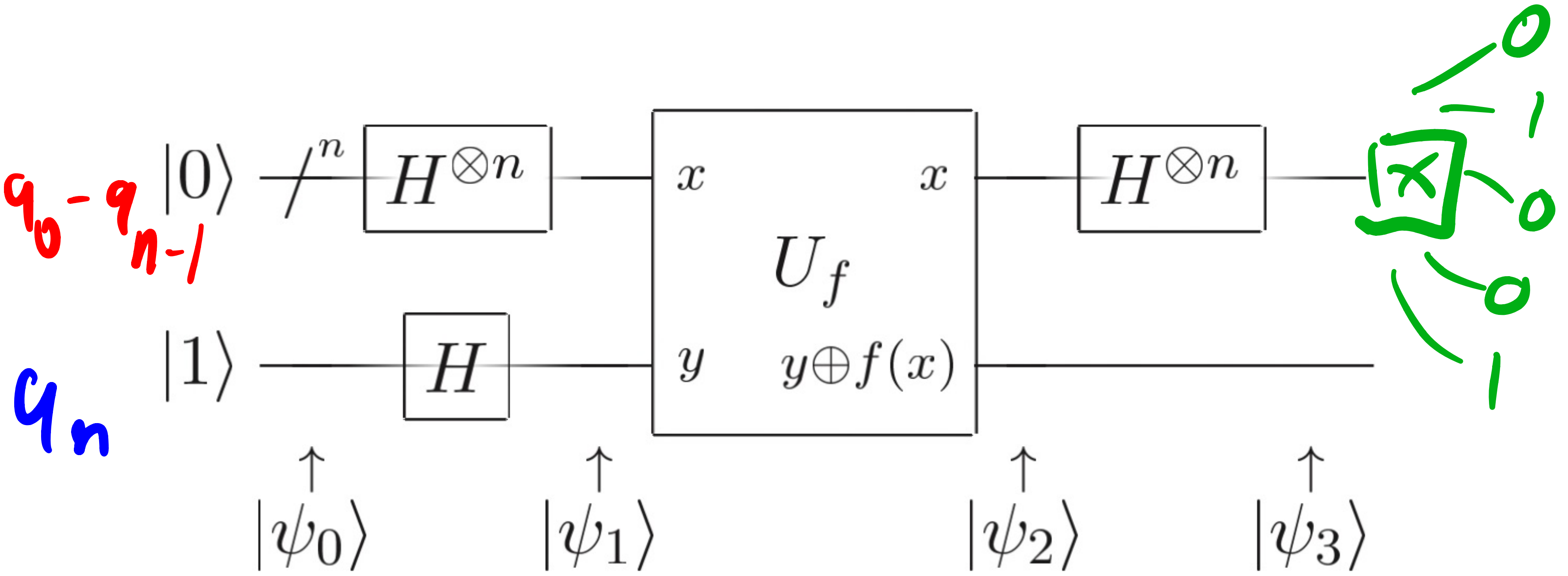


$$H \frac{1}{\sqrt{2^n}} \sum_x (-1)^{f(x)} |x\rangle$$

Example

$$= H|+ - + + - - + \cdots\rangle$$

$$= |1\,0\,1\,0\,0\,1\,1\,0 \cdots\rangle$$



Measuring

= 1 0 1 0 0 1 1 0 - . . - . >

if #0 = #1 balanced

if #0 = n or #1 = n constant

else no useful info (as far as I know)